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The Discrete Infinite

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Abstract

The author of this manuscript dedicated 5 years of trial and error in the making of a resolution of the problem of boolean classification. Basically, any SAT instance describes a unique subset of $\{0,1\}^{\mathbb{N}}$, where $\mathbb{N}$ designs the natural integers. The purpose of this manuscrit, is to offer a discrete collection of $\{0,1\}$ as the only axiom. It is possible to prove that $\mathbb{R}$ is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$. Which is also true of $\mathbb{R}^2 \simeq \mathbb{C}$. It only takes a discrete amount of computation to claim that π can be seen as collection of integers. The intuitive proof is about saying $\pi = \{3, 14, 159...\}$ increasing digit by digit to avoir redundancy. This is conter intuitive considering in the literature we consider that the 'size' of $\mathbb{R}$, is $\aleph_1$. But here comes the twist, assume for one second, that $\mathbb{R}$ is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$, $\mathbb{R}^2$ is also isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$. We will try and succeed in building a theory that will destroy the misconceptions of what I've been told. Then again Axiom theory is a complex subject, so I will do my best to stick to the knowledge gathered on the past 5 years.

1 Axioms

1.1 What is interesting in the ZFC Axioms

It works, and it works well, but if you have to summarize from a non-mathematical standpoint, there are still a lot of problems that arise. The ZF stands for Zermelo-Frænkel, C stands from the choice Axiom, under the assumption that you can actually use the Choice Axiom which let's you do as many (infinite) choices as you want, Banach-Tarski claim that you can translate a sphere into two spheres. Some people think of this as why it is not necessary to include the choice axiom.

1.2 The limitations of ZFC

We know since Gödel, that there will be undecidable properties no matter the axioms. So let's take a moment to reflect on what human being are and will be capable of in the upcoming years. What's the only certainty we have in a finite world? You can always count and add one, which means that a good theory should stand on this only prerequisite.

1.3 Two Axioms to generate a consistent Theory

The two axioms are as follow:

- Natural Integers exist, and can be manipulated as desired, ie: you can make any subset of integers (being finite or infinite).
- You are always allowed to make a discrete number of choices, so long it's about counting to infinity.

2 Construction of Traditional Sets From the Literature

2.1 Integers $\mathbb{Z}$ are a misconception

What is a negative integer, say, -3 if not 0 relative to 3. But considering you need to have two infinites $+\infty$ and $-\infty$. It suffice to assume the odd numbers $2n+1$ are none but n relative to 0 and negative, and even numbers are $2n$ are not but n the (positive) natural integer. Whenever you wish to compute some negative integers, use odd, and even for positive.

2.2 Real Numbers $\mathbb{R}$ are a misconception

Take a real number x, the absolute value $|x|$, and the floor operator $\lfloor x \rfloor$.

$$x = (-1)^{[x<0]} \sum_{k \in \mathbb{Z}} [\lfloor |x|/2^k \rfloor \mod 2 = 1] 2^k$$

Which means that there is a bijection between $\mathbb{R}$ and a subset of $\{0,1\}^{\mathbb{N}}$, which is rigorously equivalent to $\mathcal{P}(\mathbb{N})$ if you consider the $0-1$ to be the absence / presence of the integer in the subset. If you need a rigorous proof of this intuition, you can take base 2 decomposition of the real number x you are considering, and translate the $0-1$ into $1-2$ and write the associated 1 digit, 2 digit, 3 digit... integer to a set.

$$\pi = \{2, 12, 222, 2221, 22222, 2122111....\} \subset \mathbb{N}$$

2.3 Why is it bad

$$A, B \in \{0,1\}^{\mathbb{N}},\ A \times B \in \{0,1\}^{\mathbb{N}}$$

3 Why there is a unique infinite $\aleph$

3.1 Hilbert Hotel and $\mathbb{R}^n$

First of all because of the above property, it suffice to take odd/even one element from A, B, respecting the order, to claim that using $\aleph$ the discrete infinite, we can build any set and subsets in the form $\mathbb{R}^n$.

3.2 Implications on linear algebra

Given a pair of real matrices with n^2 coefficients each, the product operator.

$$\prod : (A, B) \mapsto A \cdot B$$

Is nothing but a subset of $\mathbb{R}^{3n^2}$ which is nothing but a subset of $\{0,1\}^{\mathbb{N}}$.

3.3 A note on this claim

This is way beyond my reach of understanding, but I can do finite computations with high precision on my computer. This is called a boolean decomposition algorithm, and it generates a .cnf that fits the training data with 100% accuracy, in quadratic time.

4 What is an operator?

4.1 What is a function?

Let $f : X \to Y$ be a function. From the above we know that X, Y are isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$ but did you know that using then strategy we can actually claim that $f : X \to Y$ is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$. The idea is as follows:

$$f = \{(x, f(x)) \mid x \in X, f(x) \in Y\}$$

Considering we can encode, $x, f(x)$ as elements of $\{0,1\}^{\mathbb{N}}$. And using the Hilbert Hotel Principle, we can have even $\{0,1\}$ to encode x and odd $\{0,1\}$ to encode $f(x) = y$ we can claim that f is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$.

4.2 What is an operator?

Let's now consider an operator $\odot$, using the above idea, we can claim that $X = X_1 \times X_2$ and $\odot : X_1, X_2 \to Y$.

$$x_1 \odot x_2 = y$$

We are also describing a subset of $\{0,1\}^{\mathbb{N}}$.

4.3 What is an order relationship?

An order relationship $\mathcal{R}(x_1, x_2)$ is none but an operator, that sends on a $\{0,1\}$. Thus an order relationship is also a subset of $\{0,1\}^{\mathbb{N}}$.

5 Is it possible to construct a set bigger than $\{0,1\}^{\mathbb{N}}$?

5.1 $\mathbb{R}^{\mathbb{N}}$

First thing that comes to mind.

$$\mathbb{R}^{\mathbb{N}}$$

A sequence of real numbers. We know that real numbers are isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$. The question that we are going to ask is as follows, is there a sequence of real numbers that cannot be defined by recursion on all of the previous values of the sequence. Because so long you are allowed to claim:

$$u_{n+1} = f(u_n, u_{n-1}, u_{n-2}, \ldots, u_0)$$

You can claim that your sequence of real numbers is actually nothing but the function that defines it by recursion, which by the above is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$. Let $(u_n)_n \in \mathbb{R}^{\mathbb{N}}$ be such that there is no function that can describe it from the previous occurences. Assume you wish to express any u_k for $k \in \mathbb{N}$, with infinite precision. It suffice to improve the odd/even Hilbert Hotel, to a back and forth movement, $u_0, u_0, u_1, u_0, u_1, u_2, u_0, u_1, u_2, u_3$ adding one bit of information at the time. Which is enough to claim that $\mathbb{R}^{\mathbb{N}}$ is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$.

5.2 $\mathbb{R}^{\mathbb{R}}$

The set of real functions.

$$\mathbb{R}^{\mathbb{R}}$$

This one is tricky. But we are going to extensively reduce this set with a sequence of yes/no questions and consider the set defined, so long we can actually isolate any real functions from the rest. Remember a couple of properties, we can using the Hilbert Hotel, make a discrete number of assumptions, and have an infinite room for the rest. This of f a real function. Is $f(0) = x$? can be formalized as the first zero one, and then the binary decomposition of x on the odd integers $\{0,1\}$. You can ask a finite number of questions like this. Use the limit to test continuity. And so on. But I cannot conclude at the time of writing if this suffice to claim that $\mathbb{R}^{\mathbb{R}}$ is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$. Note that $\mathcal{C}^0(\mathbb{R})$ the set of continuous real functions is isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$ using this logic.

5.3 Why it does and doesn't really matter

Assume for a second that there is a set that is not isomorphic to a subset of $\{0,1\}^{\mathbb{N}}$. It means that it cannot be described by a discrete collection of questions, no matter the questions. So the practicality of being capable of describing such a set in the human world, is close to zero.

I took some time today to improve the work I had started, it might be useful to work on probability spaces in order to find a counter example, to my claim.

$$x \mapsto \Pr(X \leq x)$$