$\int_0^1 f = \mathbb{E}(f(\mathcal{C}(\mathcal{G}(\varepsilon))))$ Integration is Nothing but a Discrete Sum

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Abstract

In this short article we will introduce Riemann Integration of a function on $(0,1)$ using the Collatz Bijection $\mathcal{C}$, which claims that there exists some sort of bridge between natural integers and real numbers in $(0,1)$. The purpose of the Geometric Law $\mathcal{G}(\varepsilon)$ with a small probability of success ε is to mimic the limit uniform law on $\mathbb{N}$. The main result of the article is as follows.

Let f be Riemann Integrable on $(0,1)$, let $D > 0$ be the precision parameter:

$$\exists \varepsilon > 0, \quad \left| \int_0^1 f(x)\mathrm{d}x - \sum_{n=1}^{\infty} \varepsilon f(\mathcal{C}(n))(1-\varepsilon)^n \right| < 2^{-D}$$

With an upper bound on ε under certain conditions.

On a more philosophical note, it means that continuous analysis is discrete, and even finite if you neglect the remainder, and only consider an approximation. A higher result would be to claim, that no matter D, $\exists \varepsilon, N > 0$ such that:

$$\left| \int_0^1 f(x)\mathrm{d}x - \sum_{n=1}^{N} \varepsilon f(\mathcal{C}(n))(1-\varepsilon)^n \right| < 2^{-D}$$

1 Introduction to my understanding of Riemann Integration

The idea behind Riemann integration lies in a very elegant formula.

$$\inf_{n \geq 1} \left\{ \left| \int_0^1 f(x)\mathrm{d}x - \frac{1}{n} \sum_{k=0}^{n-1} f(k/n) \right| \right\} = 0$$

Which implies that an integral from 0 to 1 of a function, is none but the average of its values. The problem with working on the average is that, you sometimes need a lot of samples to generate a coherent value, yet, again, the problem of having an asymptotical value at k/n. Making the series hardly integrated on say $\int_0^1 P(x)^{-2}\mathrm{d}x$ with lots of roots in $\mathbb{Q}\cap(0,1)$. There are actual tricks to challenge this method of integration. But it needs knowledge on the asymptotical behavior of the function, in order to consider it a Dirac, which ultimately leads to the Riemann Integration as the average of non infinite terms, plus a Dirac coefficients, collection.

2 Introduction to my Understanding of Measure Theory

I know probabilities are measure, and I also know that the measure $\mu_0(x) = (x \in \mathbb{N})$ exists, but cannot be interpreted as a probability measurement for the simple reason the $\mathbb{N}$ is the discrete infinite $\aleph$. Basically, suppose you have finite ressources, and want every integers to have an equal part, it has to be 0, which makes no sense because the series of 0 is zero, and not 1 as would discrete probability suggest. So let's make a thought experiment and try to imagine what should be the closest thing to this uniform measurement might be. You give people a lot, in fact everything on the probability that the person succeeded an impossible task, and that no previous person already made it. We call that probability ε, you can think of it as winning the Million dollar, every week for 52 consecutive weeks. Two things to know about it, it's possible, and it is very small. $\mu(x) = (x \in \mathbb{N})\varepsilon(1-\varepsilon)^x$, this is what we call a Geometric Law, of probability of success ε and the closest thing we know to communism on the premise that lottery ticket were free. In fact, you can prove that:

$$\mu_0 - \frac{\mu}{\varepsilon}$$

Is in physics what we call a $o(1)$.

3 Introduction to my understanding of the Collatz Conjecture

It all starts with a very simple but elegant equation.

$$C(x) = \frac{1}{2}((x \equiv 1[2])(3x+1) + (x \equiv 0[2])x)$$

And then a twisted mind of a scientist who spent way too long documenting the subject to claim that:

$$\mathcal{C}(x) = \frac{1}{2}\sum_{k=0}^{\infty}(C^k(x) \equiv 1[2])2^{-k}$$

Is both:

- Defined on $\mathbb{N}$, natural integers, zero being zero.
- A Bijection from $\mathbb{N}$ to $[0, 1)$ which can be easily disproven if you can prove Collatz conjecture to be True.

It is what we call a computational bijection with very interesting properties.

4 How it is all related

It can be summarized in simple terms: $\int_0^1 f(x)\mathrm{d}x$ is a measurement $\mu_1(f)$ evaluated in a continuous motion. But that continuous motion, can be actually emulated by taking random samples uniform and increase the number of samples. It suffice to prove that $\mathcal{C}(\mathcal{G}(\varepsilon))$ converges to $\mathcal{U}(0, 1)$. The core of the proof is that for fixed arbitrary $a < b$.

$$\Pr(a \leq X \leq b) = b - a + O(\varepsilon)$$

Which is trivial to prove, when you know that:

- $\forall a < b, C^a(x + 2^b) \equiv C^a(x)[2]$
- $C^a(x + 2^a) \equiv 1 + C^a(x)[2]$

There is a bijection between $\{1, 2, 3, ..., 2^n\}$ and $\{0, 1\}^n$ through Collatz, and it's 2^n periodic.

5 Rigorous Proof of Statements

Theorem 1.

$$\forall N > 0, \forall t \in (0, 1), \exists n,\ 2^N \geq n \geq 1, |\mathcal{C}(n) - t| \leq 2^{-N}$$

Proof. There is a 2^n periodic bijection for all n-digits up to N, which is sufficient to claim that modulo 2^{-N} the two quantities will be equal.

Proof by Python. Under the assumption that $\arctan(1) = \pi/4$ and $\arctan(0) = 0$

```
def f(x):
    return 1/(1 + x**2)

def collatz(x, prec=256, show=False):
    y = 0
    while prec:
        if x % 2:
            y += 2 ** (prec - 257)
        x = ((x % 2) * (3 * x + 1) + (x % 2 == 0) * x) / 2
        prec = prec - 1
    if show:
        print(x,y)
    return y

def approx_pi(N, eps=0.0001):
    pi = [0]
    for k in range(1, N):
        print(f" pi is approximatively {4 * sum(pi)}", end="            \r")
        pi += [eps * f(collatz(k)) * ((1 - eps) ** k)]
    print(f" pi is approximatively {4 * sum(pi)}\titerations {N}")
    return sum(pi) * 4

approx_pi(100000)
```